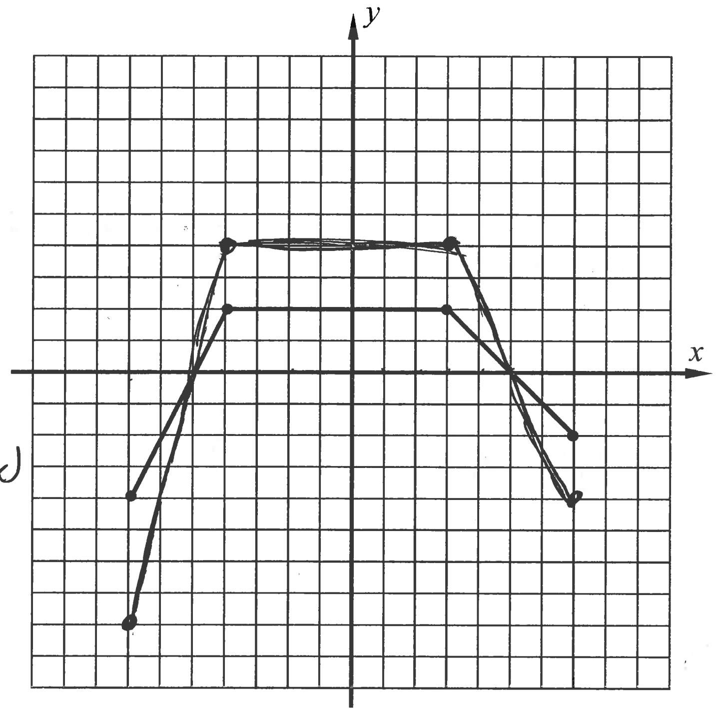
~~401~~ ~~4665~~ 467

Name: ______________________________ Date: ________________

FUNCTION TRANSFORMATIONS
COMMON CORE ALGEBRA I

We have transformed many functions this year by **shifting them** and **stretching them**. These transformations occur on a general basis and we will explore them in the next two lessons by looking almost exclusively at functions defined graphically. Still, we will rely heavily on function notation.

Exercise #1: The function $y = f(x)$ is defined by the graph below. Answer questions based on this definition. Selected points are marked on the graph.



(a) Evaluate each of the following:

$f(3) =$ 2 $\qquad$ $f(7) =$ −2

$f(-4) =$ 2 $\qquad$ $f(-7) =$ −4

(b) State the zeroes of $f(x)$.

(−5,0) & (5,0)

(c) Why is it impossible to evaluate $f(9)$?

Because 9 is not in the domain of f(x) (no y value there!)

(d) State the domain and range of $f(x)$.

Domain: $-7 \le x \le 7$ $\quad$ [−7, 7]

Range: $-4 \le y \le 2$ $\quad$ [−4, 2]

O.k. Now that we have a bit of a feel for $f(x)$ we are going to start to create other functions by **transforming** the function *f*.

Exercise #2: Let's now define the function $g(x)$ by the formula $g(x) = 2f(x)$.

(a) Evaluate each of the following. Show the work that leads to your answer. Remember, just follow the function's rule.

$g(-7) =$ $2 \cdot f(-7)$
$= 2 \cdot -4 = -8$

$g(-4) =$ $2 \cdot f(-4)$
$= 2 \times 2 = 4$

$g(3) =$ $2 \cdot f(3)$
$= 2 \cdot 2 = 4$

$g(7) =$ $2 \times f(7)$
$= 2 \times -2 = -4$

(b) How can you interpret the function rule in terms of the graph of $f(x)$?

g(x) has y-values that are double all the y-values in f(x)

(c) Sketch a graph of $f(x)$ on the grid above in Exercise #1. Write down points that you know are on $g(x)$ based on your answers to (a).

(−7, −8) $\quad$ (−4, 4)

(3, 4) $\quad$ (7, −4)

(d) State the domain and range of the function $g(x)$.

Domain: $-7 \le x \le 7$ or [−7, 7]

Range: $-8 \le y \le 4$ $\quad$ [−8, 4]

So, we see from the last exercise that when a function gets multiplied by a constant, all of the y-values get multiplied by the same constant. This has the effect of "stretching" a function.

VERTICAL STRETCH

If the function $g(x)$ is defined by $g(x) = k \cdot f(x)$, then the graph of g will be stretched (or compressed) depending on the value of k. If k is negative, it will also **reflect** the function across the x-axis.

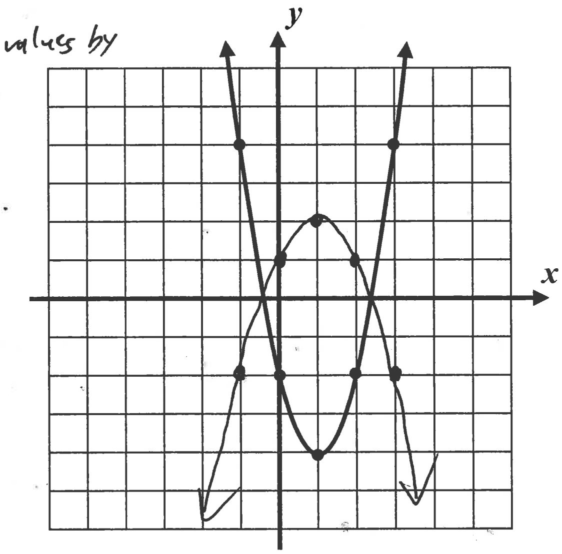
Exercise #3: A quadratic $f(x)$ is shown below. The function $g(x)$ is defined by $g(x) = -\frac{1}{2} f(x)$.

(a) Calculate the values of $g(0)$ and $g(3)$. Show your work. Explain the effect of multiplying by $-\frac{1}{2}$.

$g(0) = -\frac{1}{2} \cdot f(0)$
$= -\frac{1}{2} \cdot (-2)$
$= 1$

$g(3) = -\frac{1}{2} \cdot f(3)$
$= -\frac{1}{2} \cdot 4$
$= -2$

multiply y-values by $\frac{1}{2}$ and flip them over the x-axis.



(b) Sketch an accurate graph of $g(x)$ on the same grid as $f(x)$.

$x \leq 2$ or $(-\infty, 2]$

(c) State the range of $g(x)$.

Let's do one final problem to see how well you understand what happens to the graph of a function when it has been multiplied by a constant.

Exercise #4: The function $f(x)$ is graphed as the bold curve shown below. Three other functions are all defined in terms of f and are graphed as well. Label each curve with the appropriate function.

$g(x) = \frac{1}{2} f(x)$

$h(x) = -f(x)$

$k(x) = 2f(x)$