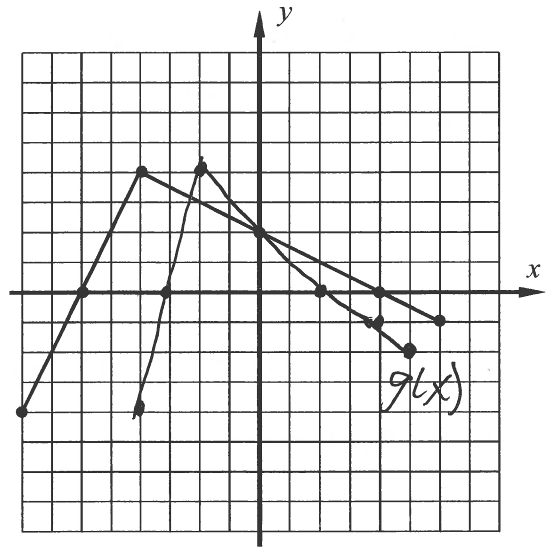
~~405~~ Alg 47 (

Name: ______________________________ Date: ________________

HORIZONTAL STRETCHING OF FUNCTIONS
COMMON CORE ALGEBRA I

In the last lesson we saw how multiplying a function by a constant stretched (or compressed) the function's outputs, and thus its graph. This was a **vertical stretch** because it only affected the vertical (output) component of the function for a given input. In today's lesson, we will see what happens to a function when you first manipulate its input.

***Exercise* #1:** The function $f(x)$ is shown on the graph below. Selected points are shown as reference. The function $g(x)$ is defined by $g(x)=f(2x)$. Notice that the multiplication by 2 happens **before** f is even evaluated. This is tricky!



(a) Find the values of each of the following. Carefully follow the rule for $g(x)$ and show your work.

$g(2)=$ f(2(2))=f(4) =0 $g(3)=$ f(2·3)= f(6) = -1

$g(-2)=$ f(2(-2))=f(-4) = 4 $g(-4)=$ f(2(-4))=f(-8) = -4

$g(0)=$ f(2·0) = f(0) =2 $g(-3)=$ f(2(-3)) = f(-6) = 0

(b) Given the definition of $g(x)$, why can we **not** find a value for $g(4)$? Explain. Because f(4)=f(8), which is undefined. 8 is not in the domain of f.

(c) State points that must lie on the graph of $g(x)$ based on your work in (a).

(2, 0) (3, -1)
(-2, 4) (-4, -4)
(0, 2) (-3, 0)

(d) Graph the function $g(x)$ based on your work from (b). Then, state the domain and range of both the original function, $f(x)$ and our new function $g(x)$. What remained the same? What changed?

Original Function: $f(x)$ **New Function:** $g(x)$

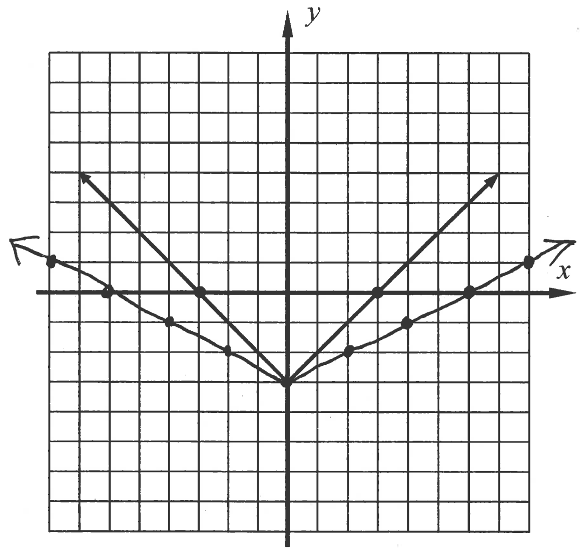
Domain:	Range:	Domain:	Range:
[-8, 6]	[-4, 4]	[-4, 3]	[-4, 4]
-8 ≤ x ≤ 6	-4 ≤ y ≤ 4	-4 ≤ x ≤ 3	-4 ≤ y ≤ 4

(e) Describe what happened to the graph of $f(x)$ when we multiplied the function's input by 2.

It got compressed (shrunk) horizontally towards the y-axis by a factor of 2.

Notice how the **horizontal stretch** worked almost counter to what we would have thought. In other words, when we multiplied the x-value by 2, it **compressed** our graph by a factor of 2. The opposite would also occur.

Exercise #2: The function $f(x)=|x|-3$ is shown on the graph below. The function $g(x)$ is defined by the formula $g(x)=\left|\frac{1}{2}x\right|-3$.



(a) Use your graphing calculator to produce a table of values for $g(x)$ and graph it on the grid to the right.

x	-8	-6	-4	-2	0	2	4	6	8
y	1	0	-1	-2	-3	-2	-1	0	1

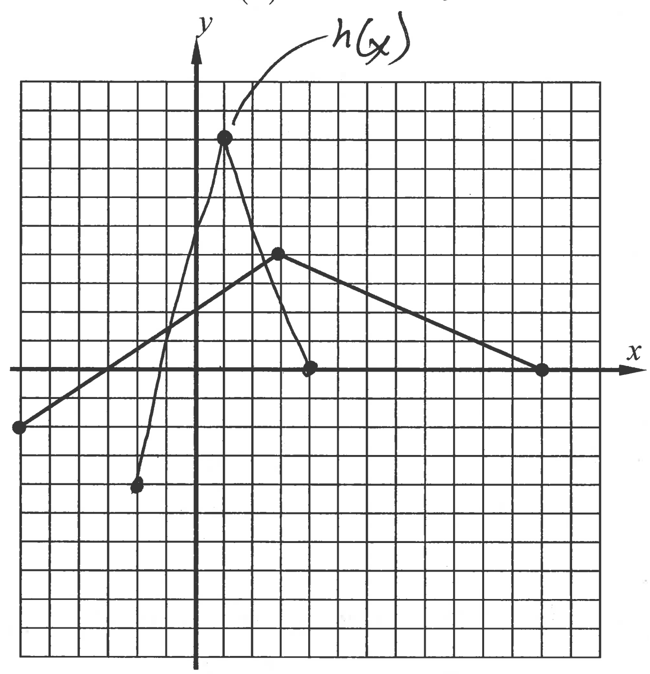
(b) What was the effect on the graph of $f(x)$ when we multiplied the input by $\frac{1}{2}$?

It gets stretched away from the y-axis by a factor of 2.

We can certainly combine the effects of both a vertical stretch and a horizontal stretch. This is harder, but if you can identify the various transformations, then the new function's graph can often be produced from the older function's fairly easily.

Exercise #3: The graph of $f(x)$ is shown on the grid below. A new function $h(x)$ is defined by:

$$h(x)=2f(3x)$$



(a) Evaluate $h(1)$. What point must lie on the graph of $h(x)$ based on this calculation? $h(1)=2\cdot f(3\cdot 1)=2\cdot f(3)$
$=2\cdot 4=8$

(1, 8)

(b) Describe the transformations that must be done to the graph of $f(x)$ to produce the graph of $g(x)$.

vertical stretch by a factor of 2

Horizontal compression by a factor of 3

(c) Graph $g(x)$ by plotting the three major points.

$(-6,-2)$ $(3,4)$ $(12,0)$

$\downarrow$ $\downarrow$ $\downarrow$

$(-2,-4)$ $(1,8)$ $(4,0)$