

## HORIZONTAL STRETCHING OF FUNCTIONS COMMON CORE ALGEBRA I



In the last lesson we saw how multiplying a function by a constant stretched (or compressed) the function's outputs, and thus its graph. This was a **vertical stretch** because it only affected the vertical (output) component of the function for a given input. In today's lesson, we will see what happens to a function when you first manipulate its input.

**Exercise #1:** The function  $f(x)$  is shown on the graph below. Selected points are shown as reference. The function  $g(x)$  is defined by  $g(x) = f(2x)$ . Notice that the multiplication by 2 happens **before**  $f$  is even evaluated. This is tricky!

- (a) Find the values of each of the following. Carefully follow the rule for  $g(x)$  and show your work.

$$g(2) =$$

$$g(3) =$$

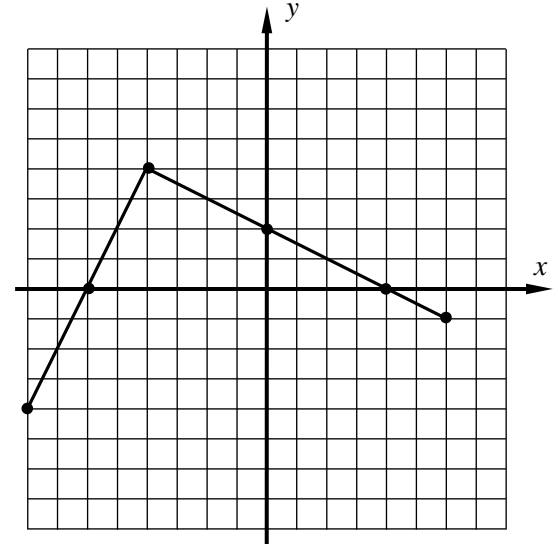
$$g(-2) =$$

$$g(-4) =$$

$$g(0) =$$

$$g(-3) =$$

- (b) Given the definition of  $g(x)$ , why can we **not** find a value for  $g(4)$ ? Explain.



- (c) State points that must lie on the graph of  $g(x)$  based on your work in (a).

- (d) Graph the function  $g(x)$  based on your work from (b). Then, state the domain and range of both the original function,  $f(x)$  and our new function  $g(x)$ . What remained the same? What changed?

**Original Function:**  $f(x)$

**New Function:**  $g(x)$

Domain:

Range:

Domain:

Range:

- (e) Describe what happened to the graph of  $f(x)$  when we multiplied the function's input by 2.



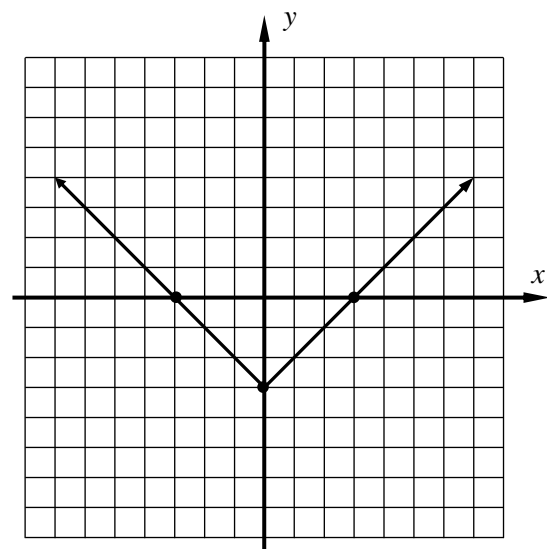
Notice how the **horizontal stretch** worked almost counter to what we would have thought. In other words, when we multiplied the  $x$ -value by 2, it **compressed** our graph by a factor of 2. The opposite would also occur.

**Exercise #2:** The function  $f(x) = |x| - 3$  is shown on the graph below. The function  $g(x)$  is defined by the

formula  $g(x) = \left| \frac{1}{2}x \right| - 3$ .

- (a) Use your graphing calculator to produce a table of values for  $g(x)$  and graph it on the grid to the right.

$x$									
$y$									



- (b) What was the effect on the graph of  $f(x)$  when we multiplied the input by  $\frac{1}{2}$ ?

We can certainly combine the effects of both a vertical stretch and a horizontal stretch. This is harder, but if you can identify the various transformations, then the new function's graph can often be produced from the older function's fairly easily.

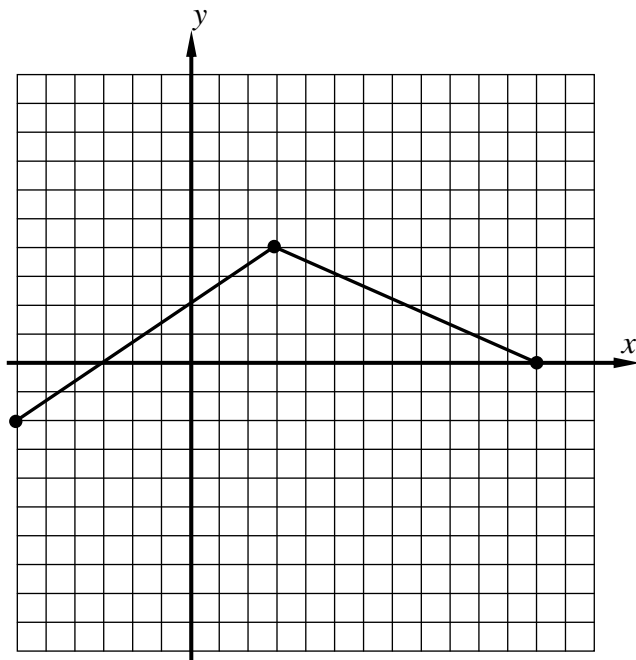
**Exercise #3:** The graph of  $f(x)$  is shown on the grid below. A new function  $h(x)$  is defined by:

$$h(x) = 2f(3x)$$

- (a) Evaluate  $h(1)$ . What point must lie on the graph of  $h(x)$  based on this calculation?

- (b) Describe the transformations that must be done to the graph of  $f(x)$  to produce the graph of  $g(x)$ .

- (c) Graph  $g(x)$  by plotting the three major points.



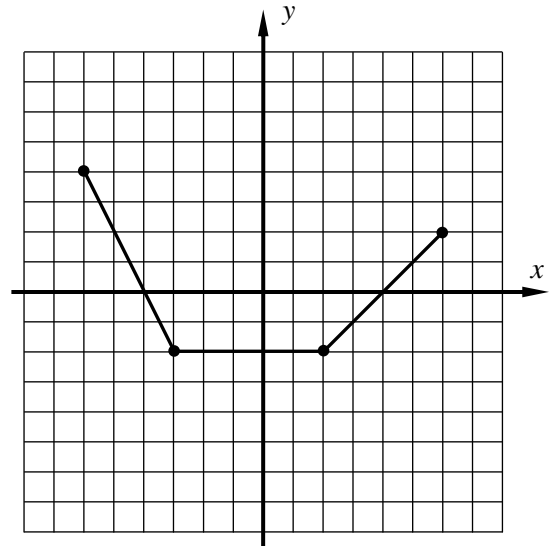
## HORIZONTAL STRETCHING OF FUNCTIONS COMMON CORE ALGEBRA I HOMEWORK

### FLUENCY

1. The function  $f(x)$  is shown graphed on the axes below with selected points highlighted. Two additional functions are defined as:

$$g(x) = f(2x) \quad \text{and} \quad h(x) = 2f(x)$$

Graph both  $g(x)$  and  $h(x)$  on the same grid and label them.



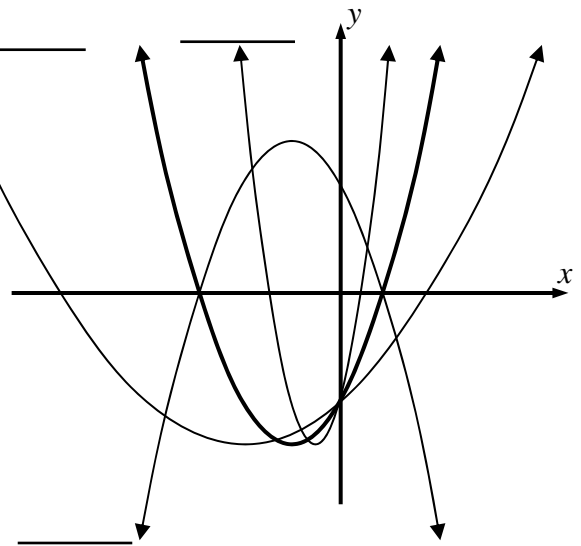
State the domain of  $g(x)$  only:

2. The quadratic function  $f(x)$  is shown graphed to the right. Three other functions are defined below with equations based on  $f(x)$ . Label each graph with its appropriate function.

$$g(x) = -f(x)$$

$$h(x) = f(2x)$$

$$k(x) = f\left(\frac{1}{2}x\right)$$



3. Which of the following formulas would indicate that the graph of  $h(x)$  was stretched in the horizontal direction by a factor of 3?

(1)  $h(3x)$

(3)  $h(x) + 3$

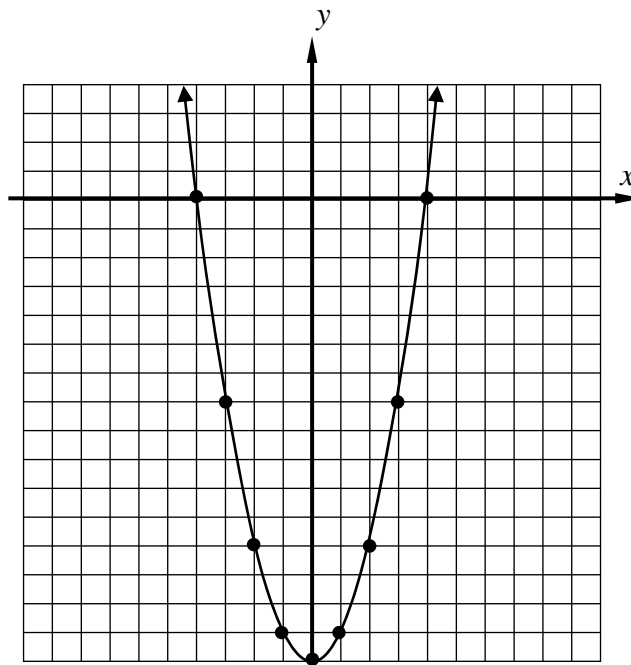
(2)  $h\left(\frac{1}{3}x\right)$

(4)  $3h(x)$



4. The parabola  $f(x) = x^2 - 16$  is shown graphed on the grid below with certain points highlighted. The function  $g(x)$  is given by  $g(x) = f(2x)$ .

- (a) What is the range of the function  $f(x)$ ?
- (b) State the zeroes of  $f(x)$ .
- (c) The function  $g(x)$  will have the equation  $g(x) = (2x)^2 - 16$ . Using your calculator, create a graph of  $g(x)$  on the grid given.
- (d) State the zeroes of  $g(x)$ . Why does this answer make sense in light of (b)?



## REASONING

5. The function  $f(x)$  is shown below. Another function is defined by the formula:

$$g(x) = f(2x) + 3$$

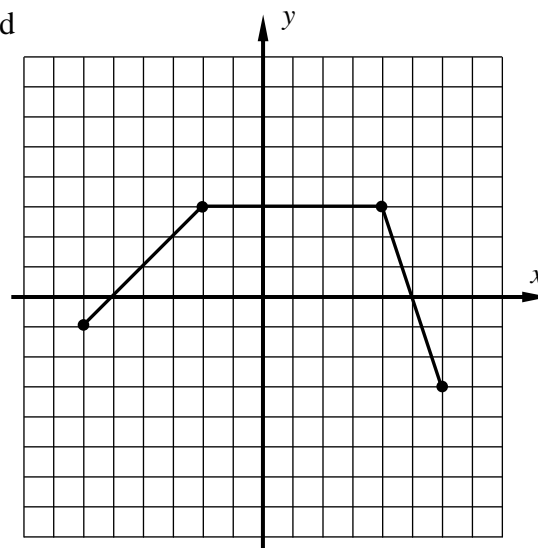
- (a) Evaluate each of the following. Show your work.

$$g(-3) =$$

$$g(-1) =$$

$$g(2) =$$

$$g(3) =$$



- (b) Plot a graph of  $g(x)$  based on (a).
- (c) What two transformations occurred to the graph of  $f(x)$  to produce the graph of  $g(x)$ ? State them and their order.

