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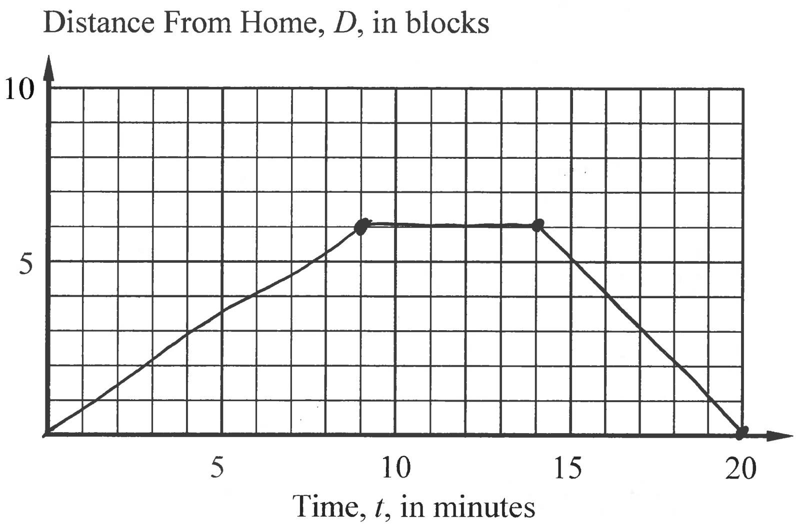
Name: ______________________________ Date: __________________

PIECEWISE LINEAR FUNCTIONS
COMMON CORE ALGEBRA I

We modeled with **piecewise functions** back in Unit #3. In today's lesson we will work specifically with **piecewise linear functions**, or those that are comprised of **linear segments**. These are particularly helpful in modeling certain situations, especially with **motion**.

***Exercise* #1:** Mateo is walking to school. It's a nice morning, so he is moving at a comfortable pace. After walking for 9 minutes, he is 6 blocks from home. He stops to answer a text on his phone from his mother. After 5 minutes standing still, he walks home quickly in 6 minutes to get a paper he forgot for school. We are going to model Mateo's distance from home, D, in blocks as a function of the time, t, in minutes since he left.

(a) Draw a graph of Mateo's distance from home on the grid provided.



(b) Determine a formula for the distance he is from home, D, over the time interval $0 \le t \le 9$.

t	D
0	0
9	6

$\frac{\Delta D}{\Delta t} = \frac{6}{9} = \frac{2}{3}$ $D = \frac{2}{3}t$ (or LinReg)

(c) Determine a formula for the distance he is from home, D, over the time interval $9 \le t \le 14$.

t	D
9	6
14	6

$\frac{\Delta D}{\Delta t} = \frac{0}{5} = 0$ $D = 0t$

(d) The trickiest part of this modeling will be to determine the linear equation for the distance, D, on the time interval $14 \le t \le 20$. Pick two points on this line and form an equation in the form $D = mt + b$.

~~LinReg~~ Using $(14, 6)$ and $(20, 0)$

By hand

t	D
14	6
20	0

$\frac{\Delta D}{\Delta t} = \frac{-6}{6} = -1$

$D = -t + b$
$0 = -20 + b$
$20 = b$
$\boxed{D = -t + 20}$

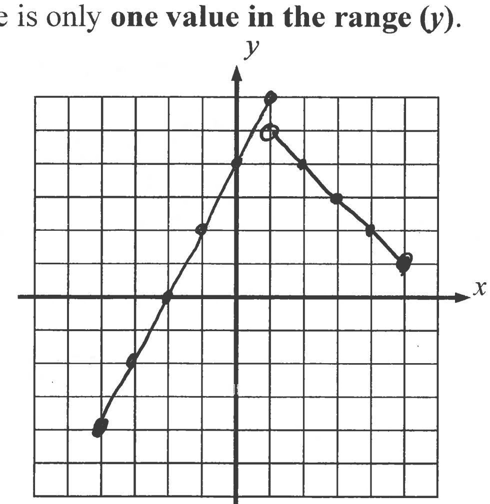
LinReg $(ax+b)$
$D = -t + 20$

Piecewise linear functions are more **complex function rules**. One way or another, though, they fit the standard definition of a function, i.e. for **every value in the domain (*x*)** there is only **one value in the range (*y*)**.

***Exercise* #2:** Consider the function defined by:

$$f(x) = \begin{cases} 2x + 4 & -4 \le x \le 1 \\ 6 - x & 1 < x \le 5 \end{cases}$$

(a) Graph the function $f(x)$ by graphing each of the two lines.

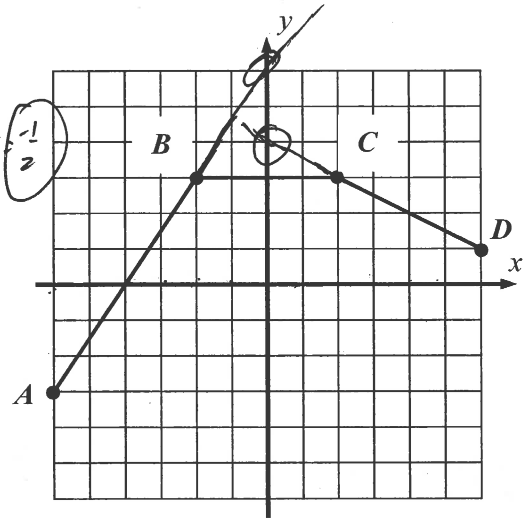


(b) State the range of the function $f(x)$.

$[-4, 6]$ or $-4 \le y \le 6$

Piecewise linear functions can often have horizontal components as well as slanted components. They will obviously never have vertical components (or they wouldn't be functions). Let's see if we can translate from a graph to a piecewise equation.

***Exercise* #3:** The piecewise linear function $f(x)$ is shown graphed below.



(a) Find the slope of each of the line segments:

$\overline{AB}$:

x	y
-6	-3
-2	3

+4, +6

$\frac{6}{4}$

$\frac{3}{2}$

$\overline{BC}$: 0

$\overline{CD}$:

x	y
2	3
6	1

$\frac{\Delta y}{\Delta x} = \frac{-2}{4}$ $\frac{-1}{2}$

(b) Now find the equation of the line that passes through each of the following pairs of points in $y = mx + b$ form where applicable. How can you find the y-intercepts by using the graph?

LinReg

$\overleftrightarrow{AB}$: $y = \frac{3}{2}x + 6$

$\overleftrightarrow{BC}$: $y = 3$

$\overleftrightarrow{CD}$: $y = -\frac{1}{2}x + 4$

(c) Write the formal piecewise definition for this function.

$$y = \begin{cases} \frac{3}{2}x + 6 & -6 \le x \le -2 \\ 0 & -2 < x \le 2 \\ -\frac{1}{2}x + 4 & 2 < x \le 6 \end{cases}$$

(d) Find the one zero of the function algebraically by setting the formula for this function that applies from $-6 \le x \le -2$ equal to zero and solving.

$\frac{3}{2}x + 6 = 0$

$\frac{3}{2}x = -6$

~~$3x = -12$~~

$x = -4$

(e) Why does setting the formula for this function that applies from $2 \le x \le 6$ equal to zero not produce a viable zero of the function?

Because the domain of $f(x)$ doesn't go far enough for that section of the graph to reach the x-axis.

(f) What parameter in the piecewise linear model indicates that the function is decreasing between $x = 2$ and $x = 6$? Explain your choice.

The slope is negative, hence the graph is decreasing.