Name: _____ Date: _____

# ANOTHER LOOK AT LINEAR AND EXPONENTIAL MODELS
## COMMON CORE ALGEBRA I

In this lesson we will be looking at linear and exponential models again and trying to understand when it makes more sense to use one, rather than the other.

***Exercise #1:*** A tank is being filled up with water. At $t = 0$, we know that the tank holds 150 gallons of water, and after one hour ($t = 1$), it holds 180 gallons of water.

(a) Assuming that the volume of water in the tank, $V$, is a linear function of time, $t$, in hours, find a formula for $V$.

$V_1(t) = 150 + 30t$

(b) By what percent did the volume of water increase from $t = 0$ to $t = 1$ ?

$\frac{30}{150} = \frac{1}{5} \rightarrow 20\%$

It increased by 20%

(c) Based on (b), write an exponential function for $V$ as a function of the time, $t$, that it has been filling.

$V_2(t) = 150(1.2)^t$

(d) After the tank has been filling for 10 hours, the volume is now at 500 gallons. Which model, the linear or exponential, better fits this data point?

$V_1(10) = 150 + 30(10) = 150 + 300 = 450$

$V_2(t) = 150(1.2)^{10} = 929$ gallons

The linear model is a better fit.

It is good to be able to look at a variety of different forms of functions and determine what type of function best fits the information. Let's take a look at that in the next exercise.

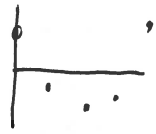
***Exercise #2:*** The area, $A$, of an oil spill is increasing and scientists are trying to model it as a function of time so that they can predict when it reaches certain critical sizes. They measure the data and find the following.

| $t$ (days) | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $A$ (square miles) | 3.5 | 4.4 | 5.5 | 6.8 | 8.5 |

(a) Explain why a linear function would not fit this data well.

LinReg(ax+b) gives $A = 1.24t + 3.26$

Residuals:



Clear curve. Linear model is not appropriate.

(b) An exponential function of the form $A = a(b)^t$ does model this data well. Select which of the following would be the most appropriate values for $a$ and $b$: ExpReg: $a = 3.5$, $b = 1.25$

| | | | | |
|---|---|---|---|---|
| $a$: | 2.6 | (3.5) | 6.4 | 8.5 |
| $b$: | 0.92 | 1.18 | (1.25) | 1.48 |

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We want to be very sure that we understand the various constants or **parameters** that come up in linear and exponential functions. Because these parameters **always** have a meaning in a physical situation.

***Exercise* #3:** Two scenarios are modeled using in (a) a linear function and in (b) an exponential function. In each case interpret the parameters that help define the functions.

(a) Plant managers at a local tire factory model the cost, $c$, in dollars of producing $n$-tires in a day by the equation: slope, y-int

$$c(n) = 6.50n + 1{,}245$$

Interpret the parameter values of 6.50 and 1,245. Include units in your answer.

The fixed costs of the factory are \$1,245 daily (no tires produced).

each tire costs \$6.50 to make

(b) Biologists model the population, $p$, of lactic acid bacteria in yogurt as a function of the number of minutes, $m$, since they added the bacteria using the equation: y-int, $1+r = 1+0.28$

$$p(m) = 135(1.28)^m$$

Interpret the parameter values of 135 and 1.28. Include units in your answer.

There was 135 bacteria in the sample at the start

Bacteria population is increasing at a rate of 28% per minute

We can also work with approximate models based on **regression work** with **bivariate data sets**.

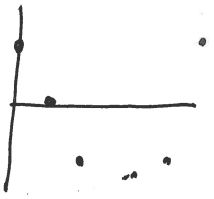
***Exercise* #4:** The rate that soil can absorb water during a rain storm decreases over time as the rain continues. The table below gives the rate $y$, in inches per hour, that water can be absorbed as a function of the number of hours that rain has been falling, $x$.

| $x$ (hours) | 0 | 1 | 3 | 5 | 6 | 10 |
|---|---|---|---|---|---|---|
| $y$ (inches/hour) | 12.3 | 9.4 | 5.9 | 3.5 | 2.7 | 1.1 |

(a) Find the linear correlation coefficient for this data set? Round to the nearest thousandth. Why is it negative? Does this indicate a strong negative correlation or a weak negative correlation? Explain.

$r = -0.938$ There is a strong, negative correlation. The absorption rate is decreasing.

(b) Produce a rough sketch of the residual plot for this data set based on (a). Does the residual plot indicate that a linear model is appropriate? Explain.

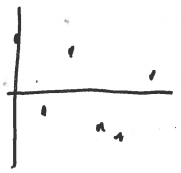


Curved residuals indicate a linear model is not appropriate.

(c) Find the exponential regression equation for this data set. Round both parameters to the nearest hundredth.

$y = 12.04(0.78)^x$

(d) Produce a rough sketch of the residual plot for this data set based on (c). Does this plot indicate a more appropriate model?



There is much more scatter in residuals. the exponential model is more appropriate.